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Heat and moisture transfer with sorption and condensation in porous clothing assemblies and numerical simulation

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Abstract

A dynamic model of heat and moisture transfer with sorption and condensation in porous clothing assemblies is presented in this paper. The model considers for the first time the effect of water content in the porous fibrous batting on the effective thermal conductivity as well as radiative heat transfer, which is a very important mode of heat transfer when there is a great difference in the boundary temperatures. The distributions of temperature, moisture concentration and liquid water content in the porous media for different material parameters and boundary conditions were numerically computed and compared. The presented numerical results showed that the condensation zone expends towards its boundaries with time.

1. Introduction

Simultaneous heat and moisture transfer in porous media is of growing interest in a wide range of scientific and engineering fields such as civil engineering, safety analysis of dam, meteorology, energy storage and energy conservation, functional clothing design, etc.

Heat and moisture transfers in porous media are coupled in rather complicated mechanisms. Although considerable previous work has been carried out on the diverse aspects of simultaneous heat and moisture transfer in the literature both theoretically and experimentally, little has been done on the coupling of heat and moisture transfer with phase change and condensation until 1980s. Ogniewicz and Tien [1] are the first workers who have contributed the analysis of the coupled heat and moisture transfer with condensation, assuming heat is transported by conduction and convection and the condensate is in pendular state. Motakef and El-Masri [2] and Shapiro and Motakef [3] extended the analysis to consider mobile condensates. Murata [4] investigated the heat and moisture transfer with condensation in a fibrous insulation slab for temperature up to 100°C both theoretically and experimentally. Murata's model considered the condensate falling under gravity and convective heat transfer. The predicted heat transfer coefficient was found to agree well with the experimental one. More recently, heat and mass transfer in wet porous media in presence of evaporation–condensation was recently revisited analytically by A. Bouddour et al. [5] using the homogenization method of asymptotic for periodic structure.

In these studies, the researchers focused mainly on materials that do not absorb moisture vapor. For textile materials, the majority of textile fibers have a certain degree of moisture absorption capability (called hygroscopicity). For instance, wool fiber can take up 38% of moisture relative to its own weight [6]. Such moisture absorption capability influences the heat and moisture transfer processes significantly as observed by previous research work [7–11]. Therefore, the theoretical models developed

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Nomenclature

C_a	water vapor concentration in the interfiber void space (kg m^{-3})
C_a^*	saturated water vapor concentration in the interfiber void space (kg m^{-3})
$C_f(x, t)$	mean water vapor concentration in a fiber over its radius at a position of the fibrous batting at a certain time (kg m^{-3})
C_f'	water vapor concentration in the fibers (it varies over the radius of the fibers) (kg m^{-3})
C_{fs}'	water vapor concentration at the surface of the fibers in the fibrous batting (kg m^{-3})
C_p	volumetric heat capacity of the fabric (kJ m^{-3} °C^{-1})
D_a	diffusion coefficient of water vapor in the air (m^2 s^{-1})
H_c	convective mass transfer coefficient (m s^{-1})
H_f	relative humidity of the fibers
H_T	convective heat transfer coefficient (kJ m^{-2} °C^{-1})
k	effective thermal conductivity of the fabric (kJ m^{-1} °C^{-1})
k_a	thermal conductivity of air filling in the fabric batting
k_e	effective thermal conductivity of the fibrous battings
L	thickness of the fabric (m)
r	radius of fibers (m)
R	resistance to direct heat transfer (i.e. 0 inner fabric, 1 outer fabric)
RH	relative humidity (%)
T	temperature of the fabric (°C)
t	real time from change in conditions (s)
w	resistance to water vapor resistance (0 inner fabric, 1 outer fabric)
W_c	water content of the fibers in the fabric (%), $W_c = C_f/\rho$
x	distance (m)

Greek symbols

ϵ	porosity of the fabric
λ	latent heat of (de)sorption or condensation of water vapor by the fibers (kJ/kg)
ρ	density of the fibers (kg/m^3)
τ	effective tortuosity of the fabric
β	an sorption constant, which is an average over all angles of incidence and is independent of position
$\lambda(x, t)$	specific heat of (de)sorption, vaporization or fusion (kJ/kg)
$\Gamma(x, t)$	the rate of (de)sorption, condensation or freezing (kg/m^3)

for non-hygroscopic materials cannot be directly adopted to describe the heat and moisture processes in hygroscopic materials.

Henry in 1939 [7] proposed a mathematical model for describing heat and moisture transfer in textiles and further analyzed the model in 1948 [8]. He derived an analytical solution by assuming that fiber moisture content is linearly dependent on temperature and moisture concentration in the air, and that fibers reach equilibrium with adjacent air instantaneously. These assumptions are too far from the actual moisture sorption process and limit the application of the model.

To improve the model, David and Nordon [10] proposed an exponential relationship to describe the rate of change of water content of the fibers, and derived a numerical solution providing space–time relationship for moisture concentration and temperature within air/fiber mass. The authors stated that their model needed further consideration of the sorption–desorption kinetics of the fibers and proper boundary conditions for practical situations, especially in evaluating the role of sorption in clothing during wear. Li and Holcombe [12] developed a new sorption rate equation that takes into account the two-stage sorption kinetics of wool fibers, and incorporated this with more realistic boundary conditions to simulate the sorption behavior of wool fabrics. They assumed that the water vapor uptake rate of the fiber consists of two components associated with the two stages of sorption identified by Downes and Mackay [13], and described by Watt [14]. Li and Luo [15,16] improved the methodology of mathematical simulation of the moisture sorption process in the fibers to derive an equation with better resolution and clearer physical meanings. The two-stage sorption process in wool fiber is simulated by a uniform diffusion equation with two sets of variable diffusion coefficients: a moisture content-dependent coefficient for the first stage and a time-dependent coefficient for the second stage. The authors further simulated and compared the theoretical results with the experimental data by using the improved vapor diffusion coefficient in fibers such as cotton, acrylic polyester and polypropylene.

Summarizing the above aspects of research on simultaneous heat and moisture transfer in porous media, we can find that the previous work has the following limitations:

1. Most of previous models assumed that the system is in a spatially steady state, i.e. temperature and moisture concentration does not change with time, but liquid water accumulates with time. Liquid water accumulation before the spatially steady state is developed was ignored.
2. The radiative heat transfer in the porous media was neglected. This is unjustifiable when the temperature difference between the boundaries are great. Several studies of glass fiber battings and of animal furs [17] have shown that radiative heat transfer can be significant in these media and becomes a main mode of heat transfer in the fibrous batting when the temperature gradients is high.
3. The thermal conductivity of the porous media is assumed to be constant. The effect of water content on thermal conductivity was ignored. However, it has been shown [18] that the effective thermal conductivity of textile fabric increases with water content and also depends on fiber's sorption properties,
4. The sorption/desorption of water by the porous media was neglected. Sorption/desorption is however an important properties of hygroscopic fibers. Wool, for example, can absorb up to 38% of water.

To simulate heat and moisture transfer mechanism in a hygroscopic fibrous batting, we shall develop a model by combining the hygroscopicity of fabric materials. In the model presented in this paper, the dynamic changes in temperature, moisture concentration, (de)sorption and water accumulation as well as the effect of water content on the effective thermal conductivity are considered. Heat transfer by conduction and radiation is built into the model. Convective heat transfer was not considered as it was shown experimentally [17] that little convective heat transfer was evident in fibrous batting having even very high porosity and low density. Numerical results of the model have provided insights on the functional design of clothing for extreme cold conditions.

The paper is composed as follows: Section 2 studies the mechanisms of coupled heat and mass transfer in porous fibrous batting and establishes the corresponding mathematical formulation; Section 3 derives and analyzes a series type analytical solutions of the model under some simplified conditions; some finite difference schemes are established and studied in Section 4 to obtain the solution of the model, and Section 5 reports on numerical results and discussion.

2. Mechanisms and mathematical formulation

In this work, we consider a model consisting of a thin inner fabric layer, a thick fibrous batting and a thin outer fabric as shown in Fig. 1. The inner fabric is close to human skin and the outer fabric is next to a cold environment. The temperature difference between the inner and outer boundaries of the model is considered as great (in the order of ~50 K).

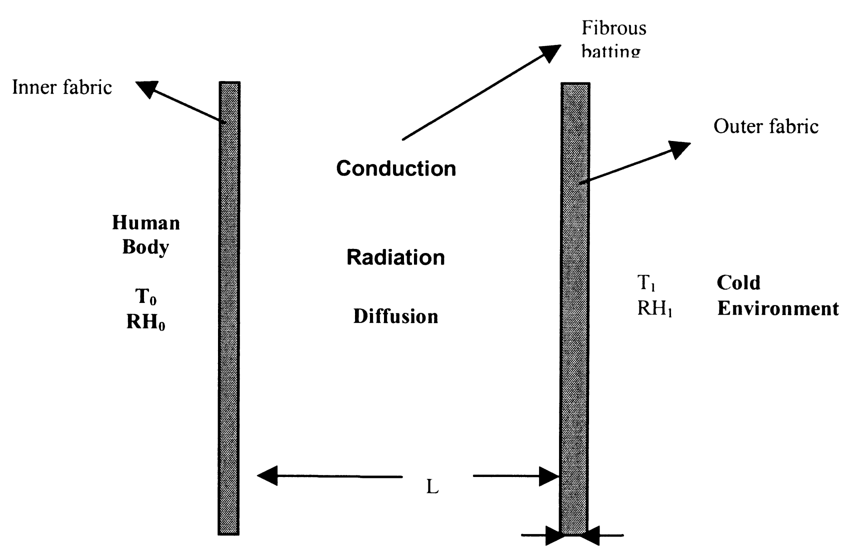

Fig. 1. Schematic diagram of the system.

2.1. Assumptions and restrictions

The following assumptions are made:

1. The fibrous batting is isotropic.
2. Local thermal equilibrium exists among all phases. This is reasonable as the pore dimension of the fibrous batting is small.
3. The liquid is immobile in the porous fabric and its effect on the material properties other than the effective thermal conductivity is negligible. This is justified when the liquid water content is small.
4. Volume changes of the fibers due to changing moisture and water content are neglected.
5. Diffusion within the fiber is considered to be so rapid that the moisture content at the fiber surface is always in sorptive equilibrium with that of the surrounding air.
6. Free convection in the porous fabric is negligible.
7. The angular distribution of radiative intensity is approximately constant. This is reasonable as the temperature difference across a penetration depth (~1 K) is much smaller than the mean temperature of the batting (~300 K). The scattering of radiation by the fibers is ignored as in the previous work [17].

2.2. Mathematical formulation of the model

Based on the above assumptions, we can establish the following mathematical equations for the coupled heat and mass transfer in the fibrous batting zone according to the conservation of heat energy and mass balance:

$$\begin{cases} C_{\mathrm{v}}(x,t)\dfrac{\partial T}{\partial t} = \dfrac{\partial}{\partial x}\left(k(x,t)\dfrac{\partial T}{\partial x}\right) \\ \quad + \dfrac{\partial F_{\mathrm{R}}}{\partial x} - \dfrac{\partial F_{\mathrm{L}}}{\partial x} + \lambda(x,t)\Gamma(x,t) \\ \dfrac{\partial F_{\mathrm{R}}}{\partial x} = -\beta F_{\mathrm{R}} + \beta\sigma T^4(x,t) \\ \dfrac{\partial F_{\mathrm{L}}}{\partial x} = \beta F_{\mathrm{L}} - \beta\sigma T^4(x,t) \\ \varepsilon\dfrac{\partial C_{\mathrm{a}}}{\partial t} + \Gamma(x,t) = \dfrac{D_{\mathrm{a}}\varepsilon}{\tau}\dfrac{\partial^2 C_a}{\partial x^2} \end{cases} \quad . \tag{1}$$

where F_{R} is the total thermal radiation incident on the element travelling to the right, F_{L} is the corresponding flux to the left, $\lambda(x, t)$ is a heat of (de)sorption, latent of heats of vaporization or latent heat of fusion. k is the effective thermal conductivity of fabric, β absorption constant, and σ the Boltzmann constant. $\Gamma(x, t)$ is (de)sorption rate, condensation rate or freezing rate. Conductive, radiative and (de)sorption heat flow are considered in the first equation in (1), and the attenuation of the radiation fluxes is given in the second and third equation in (1) (see [17]). The last equation in (1) is derived by considering the moisture accumulation by both the air and the fibers and by using the mass balance.

The heat and moisture transfer in a porous fabric is coupled by the term, $\Gamma(x, t)$, called water accumulation rate. When there is no condensation on the surface of a fiber in the porous fabric, $\Gamma(x, t)$ is of the form $\Gamma(x, t) = ((1-\epsilon)\partial C_f(x, t)/\partial t)$ and which obeys Fick's law of diffusion with a variable diffusion coefficient D_{f} depends on water content in fibers [16],

$$\frac{\partial C'_{\mathrm{f}}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(D_{\mathrm{f}}\frac{\partial C'_{\mathrm{f}}}{\partial r}\right) \tag{2}$$

where, C'_{f} is the moisture concentration in the fiber. The boundary condition around the fiber is determined by assuming that the moisture concentration at the fiber surface is instantaneously in equilibrium with the surrounding air. Hence, the moisture concentration at the fiber surface is determined by the relative humidity of the surrounding air, i.e.

$$C'_{\mathrm{fs}}(x, R_{\mathrm{f}}, t) = f(RH(x, t)), \tag{3}$$

where C'_{fs} is the moisture concentration at the fiber surface and f the functional relationship. f is a nonlinear function, which has been determined experimentally for different fibers and represented in a graph in Ref. [6]. As f cannot be expressed in a simple mathematical form, the graph was traced using a B-Spline function in the program in this work. $C_{\mathrm{f}}(x, t)$ is the mean value of $C'_{\mathrm{f}}(x, r, t)$ on $[0, R_{\mathrm{f}}]$. The relative humidity in the interfiber space can be determined by [6]

$$RH(x, t) = \frac{T(x, t) \times C_{\mathrm{a}}(x, t) \times 10^6}{216.5 \times \mathrm{Vap}(T(x, t))},$$

where the vapor pressure is obtained from temperature by the following relationship:

$$\mathrm{Vap}(T) = \begin{cases} 1013.25\mathrm{e}^{13.3185s-1.976s^2-0.6445s^3-0.1299s^4} & T \le 273.16 \\ 10^{10.5380997-2663.91/T} & T > 273.16 \end{cases}.$$

where $s = T - 273.26$.

The sorption of moisture in the porous fibrous batting can then be determined by

$$W_{\mathrm{c}}(x, t) = C_{\mathrm{f}}(x, t)/\rho,$$

where ρ is the density of the fiber.

When the relative humidity exceeds 100% at a position in the batting, condensation occurs. In the condensation region, the liquid and water vapor are in a thermodynamic equilibrium. The vapor concentration is saturated and solely determined by the temperature, i.e. $C_a^* = C_a^*(T(x, t))$. Under such circumstances, the water condensation rate $\Gamma(x)$ can be uniquely determined by the mass balance equation, i.e.

$$\Gamma(x, t) = \left(\frac{D_a}{\tau} \frac{\partial^2 C_a^*(x, t)}{\partial x^2} - \frac{\partial C_a^*(x, t)}{\partial t} \right),$$

where

$$C_a^*(x, t) = 216.5 \times \mathrm{Vap}(T(x, t)) \times 10^{-6} / T(x, t).$$

With $\Gamma(x)$, the water content can be calculated by

$$W_c(x, t) = \frac{1}{\rho} \int_0^t \Gamma(x, t)\, dt. \tag{4}$$

As mentioned before, the effective thermal conductivity of porous fibrous battings is related to the water content. Figures 2 and 3 plots the experimentally determined [18] effective thermal conductivity against water content for wool batting having a porosity of 0.915 and a polypropylene batting having the porosity of 0.87, where the regain means equilibrium water content of fiber under certain environment conditions. The effective thermal conductivity used in our computation is estimated from the conductivity of the air k_a and the effective thermal conductivity k_e given in Fig. 2 or Fig. 3 according to:

$$k = [(1 - \varepsilon)k_e + \varepsilon k_a]/c,$$

where ϵ is the porosity of the fabric batting, c is a constant for wool and polypropylene fabric, 0.085 and 0.13, respectively.

The volumetric heat capacity $C_v(x, t)$ of the fibrous batting and latent heat $\lambda(x, t)$ in non-condensation zone are also functions of water content. The equations for calculating $C_v(x, t)$ and $\lambda(x, t)$ are given in Section 5 according to Ref. [16].

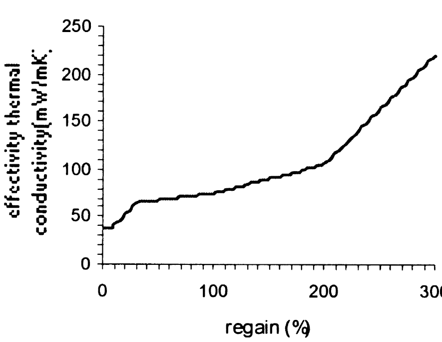

Fig. 2. The effective thermal conductivity against water content in a wool batting.

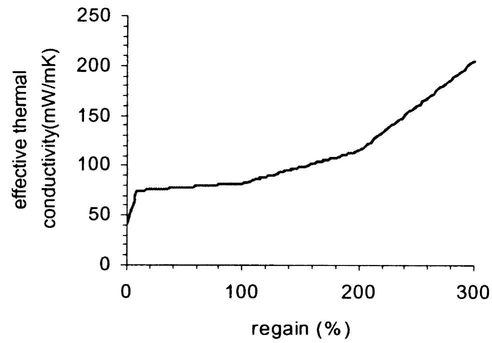

Fig. 3. The effective thermal conductivity against water content in a polypropylene batting.

3. Analytical solution under a simplified initial and boundary conditions

In order to obtain a general understanding on the solutions of the problem, an analytical solution to Eq. (1) was attempted. Since we concentrate on the temperature and vapor concentration distribution in the porous medium, we get from (1) by eliminating F_R and F_L, that

$$\begin{cases} \dfrac{\partial^2}{\partial x^2}\left(c_v \dfrac{\partial T}{\partial t}\right) - \beta^2 c_v \dfrac{\partial T}{\partial t} = \dfrac{\partial^2}{\partial x^2}\left(k \dfrac{\partial T}{\partial x}\right) - \beta^2 \dfrac{\partial}{\partial x}\left(k \dfrac{\partial T}{\partial x}\right) \\ \quad -8\beta\sigma T^3 \dfrac{\partial^2 T}{\partial x^2} + 24\beta\sigma T^2 \left(\dfrac{\partial T}{\partial x}\right)^2 + f(x, t) \\ \dfrac{\partial C_a}{\partial t} = a_3 \dfrac{\partial^2 C_a}{\partial x^2} - a_4 \Gamma(x, t). \end{cases} \tag{5}$$

where

$$\Gamma(x, t) = \frac{\partial C_f(x, t)}{\partial t}(1 - \epsilon),$$

$$f(x, t) = \frac{\partial^2}{\partial x^2}(\lambda(x, t)\Gamma(x, t)) - \beta^2 \Gamma(x, t)).$$

It can be seen from Eq. (5) that the equation concerning on temperature becomes a non-linear partial differential equation and very difficult to solve this kind of partial differential equation analytically since the radiative heat flow is involved in the model, although the concerned region of the model is regular. The analytical solution was therefore attempted under the following very simplified initial and boundary conditions where we assumed

$$\begin{cases} T_x(0,t)=\Delta T_1, \quad T_x(1,t)=\Delta T_2 \\ T(x,0)=T_0 \\ \dfrac{\partial}{\partial x}C_a(0,t)=C_{a0}, \quad \dfrac{\partial}{\partial x}C_a(1,t)=C_{a1} \\ C_a(x,0)=C_0 \end{cases}. \tag{6}$$

We also assume that there is a smaller boundary temperature difference to the porous fibrous batting, and that the coefficients k, C_v and λ are constants. Let

$$T(x,t)=T_0+T_1(x,t)$$

and

$$T^4(x,t)\approx T_0^4+4T_0^3T_1(x,t).$$

Let $A=((\Delta T_2-T_1)/2)$ and $B=\Delta T_1$. We get the solution from mathematical analysis in the following form:

$$T(x,t)=Ax^2+Bx+T_0+\frac{v_0(t)}{2}+\sum_{n=1}^{+\infty}v_n(t)\cos n\pi x \tag{7}$$

where

$$v_0(t)=-\frac{4}{\beta^2}\int_0^t\int_0^1 f(x,t)\,\mathrm{d}x\,\mathrm{d}t-\left(\frac{2}{3}A+B\right)+2T_0$$

and

$$v_n(t)=$$
$$\mathrm{e}^{-c_nt}\left[a_n+\frac{2}{(n\pi)^2-\beta^2}\int_0^t \mathrm{e}^{c_n(\xi-t)}\int_0^1 f(x,\xi)\cos n\pi x\,\mathrm{d}x\,\mathrm{d}\xi\right].$$

$$a_n=-\frac{2}{(n\pi)^2}[(-1)^{n+1}2A+B((-1)^n-1)].$$

$$c_n=\frac{a_1(n\pi)^4-\tilde{a}_2(n\pi)^2}{(n\pi)^2-\beta^2} \text{ and } \tilde{a}_2=8\beta\sigma a_2T_0^3-a_1\beta^2$$

Similar to water concentration C_a, we have

$$C_a(x,t)=A_1x^2+B_1x+\frac{C_0(t)}{2}+\sum_{n=1}^{+\infty}c_n(t)\cos n\pi x \tag{8}$$

where $A_1=((C_{a1}-C_{a0})/2)$, B_1-C_{a0}, and

$$C_0(t)=2C_0-\left(\frac{2}{3}A_1+B_1\right)-2a_4\int_0^t\int_0^1\Gamma(x,t)\,\mathrm{d}x\,\mathrm{d}t$$

$$C_n(t)=\mathrm{e}^{-3(n\pi)^2t}\left[d_n-a_4\int_0^t \mathrm{e}^{a_3(n\pi)^2(\xi-t)}\times\int_0^1\Gamma(x,\xi)\cos n\pi x\,\mathrm{d}x\,\mathrm{d}\xi\right]$$

$$d_n=-\frac{2}{(n\pi)^2}[(-1)^{n+1}\times 2A_1+B_1((-1)^n-1)].$$

It can be seen from the complexity of the above series type solution, that an analytical solution to Eq. (1) is almost impossible if no simplifying assumptions are made. Furthermore, even under such simplification, it is difficult to determine the condensation region in the batting since there is no explicit expression of the term $\Gamma(x,t)$.

4. Finite difference scheme

We chose a positive integer N, and inscribed it into the strip such that $\{(x,t)\colon x\in[0,L],\ t\geq 0\}$, a rectangular grid $\{(ih,n\Delta t)|\ i=0,1,\ldots,N,\ n\geq 0\}$, where $\Delta x=(L/N)$. Denote by T_i^n and C_{ai}^n the values of the temperature and the vapor concentration in ih at time $n\Delta t$, respectively. Since $(\partial^2T^4/\partial x^4)=12T^2(\partial T/\partial x)^2+4T^3(\partial^2T/\partial x^2)$ appears in nonlinear form, we replace the first term of the right hand and T^3 in the second term by the corresponding value at previous time step, and utilize the mean difference formula and backward difference formula to x and t, respectively, to get a stable scheme. From the first part of Eq. (5), we get the following finite difference scheme:

$$\begin{aligned} &A_i^{n+1}T_{i-2}^{n+1}+B_i^{n+1}T_{i-1}^{n+1}+C_i^{n+1}T_i^{n+1}+D_i^{n+1}T_{i+1}^{n+1}\\ &\quad-E_i^{n+1}T_{i+2}^{n+1}\\ &\quad=F_i^{n+1},\\ &\quad i=2,3,\ldots,N-2, \end{aligned} \tag{9}$$

where

$$\omega = h^2/\Delta t;\ A_i^{n+1} = KK_{i-2};\ B_i^{n+1} = -KK_{i-2} - (3+\beta^2 h^2)KK_{i-1} - 8\beta\sigma h^2 (T_i^n)^3 - \omega cv_{i-1};$$

$$C_i^{n+1} = (3+\beta^2 h^2)(KK_i + KK_{i-1}) + \omega(2+\beta^2 h^2)cv_i + 16\beta\sigma h^2 (T_i^n)^3;$$

$$D_i^{n+1} = -KK_{i+1} - (3+\beta^2 h^2)KK_i - 8\beta\sigma h^2 (T_i^n)^3 - \omega cv_{i+1};$$

$$E_i^{n+1} = KK_{i+1};$$

$$F_i^{n+1} = -\omega(cv_{i+1}T_{i+1}^n - 2cv_i T_i^n + cv_{i+1}T_{i-1}^n) + \omega\beta^2 h^2 cv_i T_i^n + R_i^{n+1}$$

$$h^4 \text{ and } R_i^{n+1} = 24\beta\sigma(T_i^n(T_{i+1}^n - T_i^n))^2/h^2 - \frac{\partial^2}{\partial x^2}(\lambda(x,t)\Gamma(x,t))|_{(i,n)} + \beta^2\lambda_i^n \Gamma(x_i, t_n)$$

where $KK_i = (K_i + K_{i+1})/2 := (k(x_i) + k(x_{i+1}))/2$.

Similarly, the finite difference scheme concerning $C_a(x, t)$ is derived from the second part of Eq. (5):

$$-\frac{D_a}{\omega\tau}C_{ai-1}^{n+1} + \left(1 + \frac{2D_a}{\omega\tau}\right)C_{ai}^{n+1} - \frac{D_a}{\omega\tau}C_{ai+1}^{n+1} = C_{ai}^n + \frac{1-\varepsilon}{\varepsilon}\Gamma(x_i, t_n)\Delta t \qquad (10)$$

$$i = 1, 2, \ldots, N-1.$$

Although it is not difficult to know from the computational mathematics theory that the given finite difference schemes are stable in mathematics, the practical computation closely depends on the properties of the coupled term because of the fact that the vapor concentration in porous media is non-negative.

4.1. Boundary conditions

To get the numerical solution for the problem, four boundary conditions are needed for the equations concerning temperature and vapor concentration, respectively. The heat and moisture transfer in the inner and outer fabric are simplified since the thickness of the inner or outer fabric is very small ($\approx$0.1 mm) compared with the thickness of the fibrous batting ($\approx$40 mm). The properties of heat and moisture transfer in the boundary fabric are described by the simple resistance to heat and vapor transfer. The heat and water vapor within the inner or outer fabric are assumed to obey linear distribution.

The boundary conditions to the main partial differential equations can be established by considering convective nature of the boundary fabrics, air layers and the resistance of heat and water vapor transfer in the inner and outer fabrics. We have

$$\begin{cases} k(x,t)\dfrac{dT^{n+1}(x)}{dx}|_{x=0} = \dfrac{T_0^{n+1} - T_{b0}}{R_0} \\ k(x,t)\dfrac{dT(x,t_{n+1})}{dx}|_{x=L} = \dfrac{T_{b1} - T_N^{n+1}}{R_1 + (1/H_2)} \end{cases}. \qquad (12)$$

and

$$\begin{cases} D_a\varepsilon\dfrac{\partial C_a^{n+1}}{\partial x}|_{x=0} = \dfrac{C_{a0}^{n+1} - C_{ab0}}{w_0} \\ D_a\varepsilon\dfrac{\partial C_a^{n+1}}{\partial x}|_{x=L} = \dfrac{C_{ab1} - C_{aN}^{n+1}}{w_1 + (1/H_c)} \end{cases}, \qquad (13)$$

where R_0 and R_1 are the resistance to heat transfer in the inner and outer fabrics, respectively, and H_t the convective heat transfer coefficient between the outer surface of the outer fabric and the environment. w_0 and w_1 are the resistance to water vapor transfer of the inner and outer fabric, and H_c is the convective vapor transfer coefficient between the outer surface of the outer fabric and the environment.

At the left hand boundary, part of the radiative heat flux incident on this boundary $F_L(0, t)$ is reflected and the inner fabric radiates. These two effects add to give the flux leaving the right side of the inner fabric $F_R(0, t)$. If the emissivity of the inner fabric is ϵ_1, we have

$$(1-\varepsilon_1)F_L(0,t) + \varepsilon_1\sigma T^4(0,t) = F_R(0,t)$$

Similarly, at the right hand boundary, we have

$$(1-\varepsilon_2)F_R(L,t) + \varepsilon_2\sigma T^4(L,t) = F_L(0,t).$$

The above boundary conditions can lead to the following linear equations by applying the finite difference scheme:

$$A_1^{n+1}T_0^{n+1} + B_1^{n+1}T_1^{n+1} + C_1^{n+1}T_2^{n+1} + D_1^{n+1}T_3^{n+1} = 0 \qquad (14)$$

and

$$A_{N-1}^{n+1}T_{N-3}^{n+1} + B_{N-1}^{n+1}T_{N-2}^{n+1} + C_{N-1}^{n+1}T_{N-1}^{n+1} + D_{N-1}^{n+1}T_N^{n+1} = 0, \qquad (15)$$

where

$$A_1^{n+1} = -(1 + co_1 h)K_0 + 8\beta\sigma(T_0^n)^3 h^3;$$

$$B_1^{n+1} = (2 + co_1 h)K_1 + (1 + co_1 h)K_0 - 8\beta\sigma(T_0^n)^3 h^3;$$

$$C_1^{n+1} = -(2 + co_1 h)K_1 - K_2 \text{ and } D_1^{n+1} = K_2$$

and

$$A_{N-1}^{n+1} = -K_{N-2};$$

$$B_{N-1}^{n+1} = (2 + co_2h)K_{N-1} + K_{N-2};$$

$$C_{N=1}^{n+1} = -(2 + co_2h)K_{N-1} - (1 + co_2h)K_N + 8\beta\sigma(T_N^n)^3h^3;$$

$$D_{N-1}^{n+1} = (1 + co_2h)K_N - 8\beta\sigma(T_N^n)^3h^3;$$

$$co_1 = \frac{\beta\varepsilon_1}{2-\varepsilon_1} \text{ and } co_1 = \frac{\beta\varepsilon_1}{2-\varepsilon_2}.$$

5. Numerical solutions and discussion

In this section, numerical analysis and solutions to the system consisting of polypropylene and wool battings under given boundary conditions of temperature and relative humidities are discussed. The numerical solution simulates a person wearing the clothing and moving suddenly from an initial warm condition to a cold environment. The initial warm atmospheric condition is assumed to be 20°C and 80% RH and the cold condition is assumed to be −20°C and 90% RH. The inner fabric is assumed to be close to the human skin and the microclimate next to skin is assumed to be 33°C and 96% RH.

In computation, the values of the material parameters are listed as follows:

For polypropylene:

$\rho = 910$ kg/m^3 (density of a fiber)

$R_f = 10^{-5}$ m (radius of a fiber)

$D_f = 1.3\text{e}^{-13}$ m^2/s

(diffusion coefficient of vapor in a fiber)

$$\lambda = \begin{cases} 2522 \text{ kJ/kg} & \text{in dry region} \\ 2260 \text{ kJ/kg} & \text{in wet region} \\ (2260+333) \text{ kJ/kg} & \text{in freezing region} \end{cases}.$$

$C_v = 1715.0$ kJ/m^3 K

(volumetric heat capacity of fabric)

For wool:

$\rho = 1310$ kg/m^3 (density of a fiber)

$R_f = 1.03\text{e}^{-5}$ m (radius of a fiber)

$D_f = 6.0\text{e}^{-13}$ m^2/s

(diffusion coefficient of vapor in a fiber)

$$\lambda = \begin{cases} 1602.5\text{e}^{-11.72W_c} + 2522.0 \text{ kJ/kg} & \text{in dry region} \\ 2260 \text{ kJ/kg} & \text{in wet region} \\ (2260+333)\text{kJ/kg} & \text{in freezing region} \end{cases}.$$

$C_v = 373.3 + 4661W_c + 4.221T$ kJ/m^3 K

(volumetric heat capacity of fabric)

The following parameters are also used in the computations:

$$\varepsilon = \begin{cases} 0.915 & \text{wool} \\ 0.87 & \text{polypropylene} \end{cases}$$

(porosity of the fabric batting zone)

$\tau = 1.2$ (effective tortuosity of the fabric batting)

$K_a = 0.025w/m$

k(thermal conductivity of vapor in the battings)

$D_a = 2.5\text{e}^{-5}$ m^2/s (diffusion coefficient in the air)

$L = 0.04$ m

$e_1 = e_2 = 0.9$

For the results shown in Figs. 4–11, the resistance to heat transfer of the inner (R_0) and outer (R_1) fabric was taken to be 0.003 m^2 K/W, and the water vapor resistance of the inner (w_0) outer (w_1) fabric was taken to be 10^3 s/m (corresponding to a moderately permeable breathable fabric).

Figures 4 and 5 show the changes of temperature distribution with time in the polypropylene and wool battings, respectively. Since the hygroscopicity of wool is much higher than that of polypropylene, more water is absorbed into the wool fibers than into polypropylene fibers, and as a result, the temperature in the most part of wool batting is lower than that in polypropylene batting at the same time and location. To show this more clearly, Fig. 6 plots out the temperature distributions of the two kinds of materials at various times. It can be seen that the temperature

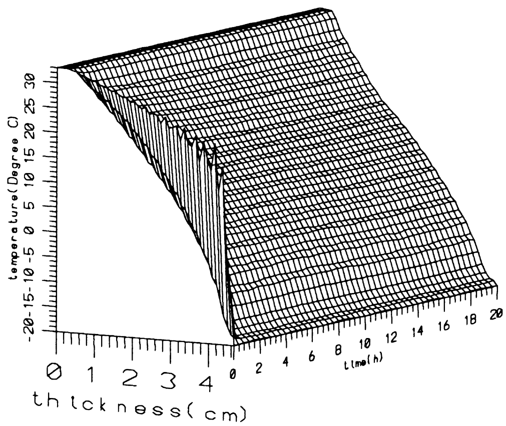

Fig. 4. Changes of temperature distribution with time in the polypropylene batting.

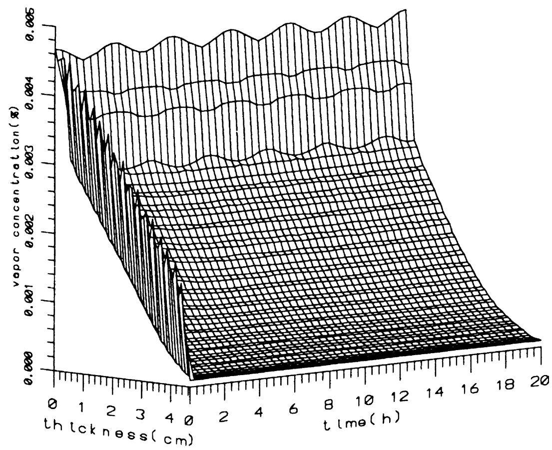

Fig. 7. Distribution of water vapor concentration in the polypropylene batting.

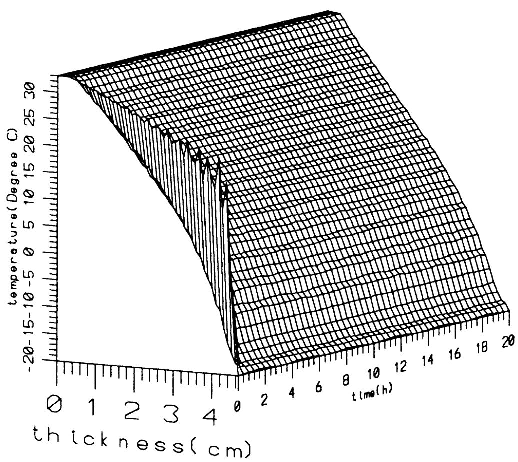

Fig. 5. Changes of temperature distribution with time in the wool batting.

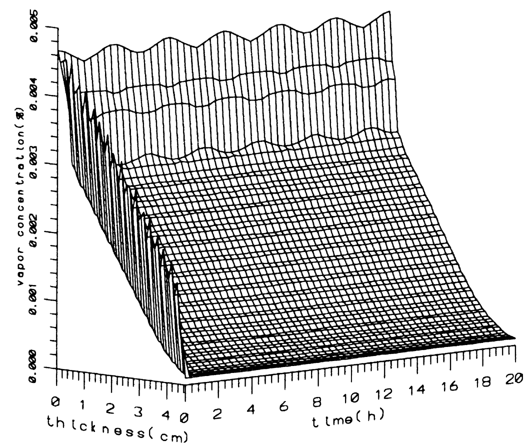

Fig. 8. Distribution of water vapor concentration in the wool batting.

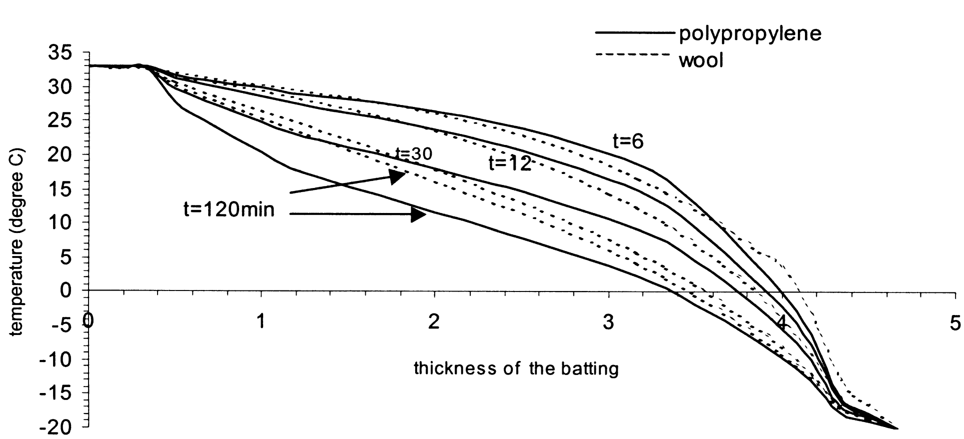

Fig. 6. Temperature distribution of polypropylene and wool batting at various times.

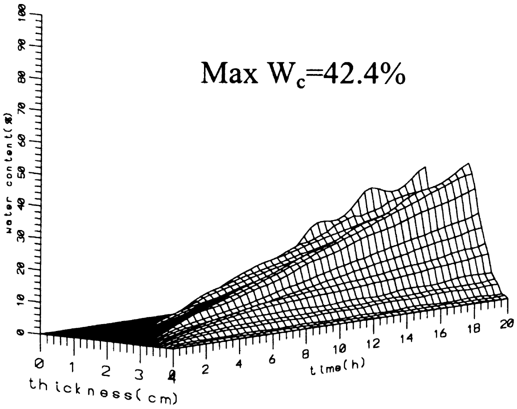

Fig. 9. Distribution of water content in the polypropylene batting.

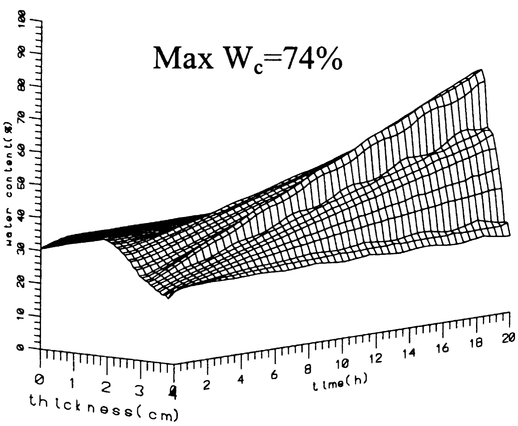

Fig. 10. Distribution of water content in the wool batting.

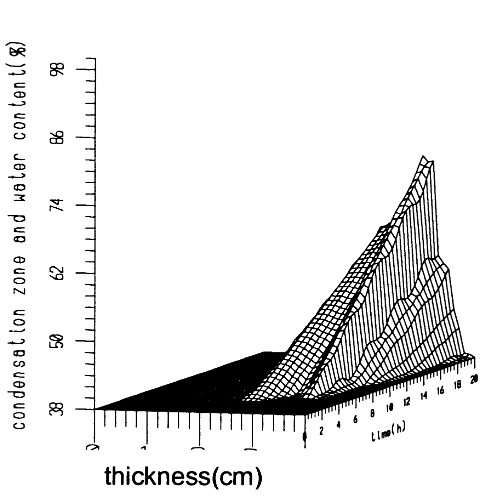

Fig. 11. Distribution of condensed water in the wool batting.

difference between the polypropylene and wool batting can be up to 7–8°C.

The distributions of water vapor concentration in the polypropylene and wool battings are given in Figs. 7 and 8, respectively.

The distribution of water content in the fibrous batting is very interesting. Figures 9 and 10 show the distributions of water content in the polypropylene and wool battings under the same conditions and parameters, respectively. It can be seen that, the water content increases rapidly in region closed to the outer side of the batting. The maximum of the water content in polypropylene and wool batting within 20 h is 42.4 and 74%, respectively. The water content can be in the form regain absorbed by the fibers or in the form of liquid or ice condensates. Since the saturated regain of polypropylene is only 0.00352%, the water content in the polypropylene batting is almost all in the form of condensed liquid water or ice, hence the distribution of the condensed water in the polypropylene batting is almost the same as the that shown in Fig. 9. However, as the saturated regain of wool batting is 38.0%, much of the water content is in the form of regain absorbed by the fibers. The distribution of the condensed water in the wool batting is shown in Fig. 11. From the results, we can see that, the water vapor in the assembly consisting of polypropylene batting begins to condense (or freeze) in a very short time, whereas condensation does not occur in the assembly consisting of wool batting until after about 4 h.

Since condensation should be avoided in clothing assembly to prevent the significant loss of thermal insulation associated with the wicking and evaporation of condensed water. Numerical experiments were carried out to examine the effects of using different inner and outer fabrics and changing the porosity of batting on the occurrence of condensation. Figure 12 shows the distribution of condensed water in the assembly consisting of polypropylene batting when water vapor resistance of the inner fabric increases to 2700 s/m (corresponding to a very impermeable coated breathable fabric) and that of outer fabric is remained unchanged at 1000 s/m (corresponding to a moderately permeable coated breathable fabric). Comparing Figs. 9 and 12, we can see that the maximum condensed water content within 20 h is reduced from 42.4% to 30.5% by increasing the water vapor resistance of the inner fabric. Figure 13 shows the distribution of condensed water content in the assembly consisting of polypropylene batting when the resistance to vapor transfer of the inner and outer fabrics are 2700 and 100 s/m (corresponding to very permeable coated breathable fabric), respectively. As can be seen, the maximum of condensed water content within 20 h is reduced further to 21.33%.

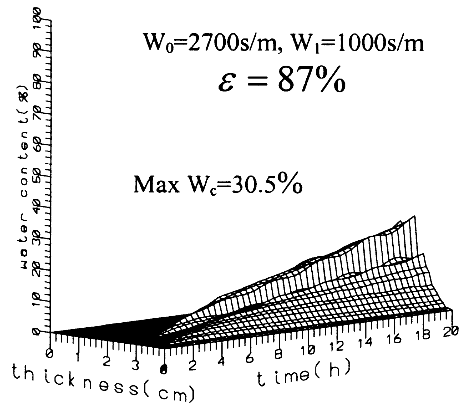

Fig. 12. Distribution of condensed water in the assembly consisting of polypropylene batting with a water vapor resistance of the inner fabric increased to 2700 s/m.

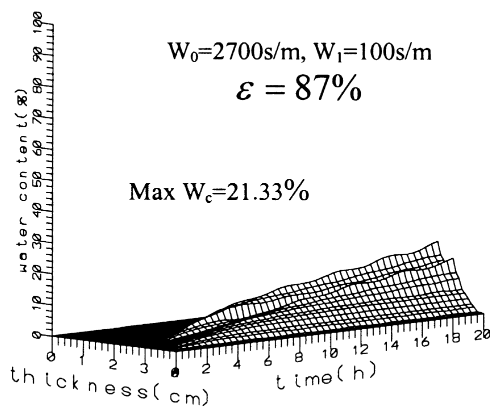

Fig. 13. Distribution of condensed water content in the assembly consisting of polypropylene batting when the resistance to vapor transfer of the inner and outer fabrics are 2700 and 100 s/m.

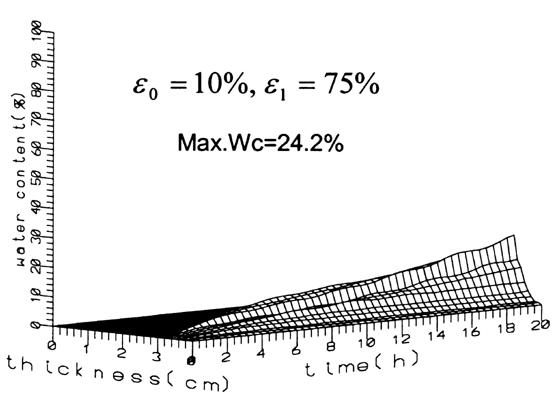

Fig. 14. Distribution of condensed water content with a changed porosity of the polypropylene fibrous batting.

The effects of changing the porosity of the fibrous batting are shown in Fig. 14. In this case, we assume that the fibrous batting consists of three kinds polypropylene fibers with various porosity, the porosity is 10% in the inner 1.4 region and 75% in the outer 1/4 region of the batting, and obeys linearly distribution within the remaining region. The resistance to vapor transfer of the inner and outer fabric is taken as 1000 and 1000 s/m, respectively. The results show that the maximum condensed water content within 20 h is about 24.2% and the condensation occurs simultaneously in the regions near the inner and outer boundaries. Comparing the results shown in Figs. 14 and 9, it can be seen that the condensation can be reduced by changing the porosity distribution of the batting. Numerical experiments have also been carried out to wool fabric batting under the same conditions. The theoretical results show that there is no condensation in the fabric batting within 20 h.

6. Conclusion

A dynamic model of heat and moisture transfer with sorption and condensation in porous clothing assemblies has been established in the paper. In this model, the important effect of water content in the porous fibrous batting on the effective thermal conductivity as well as radiative heat transfer, which is a very important mode of heat transfer when there is a great difference in the boundary temperatures, are considered for the first time. Numerical results of the model have shown that less condensation occurs in clothing assembly consisting of fibrous battings of higher hygroscopicity within a period of time. Condensation can also be reduced by increasing the water vapor resistance of the inner fabric, reducing the water vapor resistance of the outer fabric and changing the porosity distribution of the batting. It is believed that the model can not only find applications in functional clothing design, but also in other scientific and engineering fields involving heat and mass transfer in porous media.

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